Jpn. J. Appl. Phys. Vol. 38 (1999) pp. 5176–5177
Part 1, No. 9A, September 1999

Short Note

An Improved Method for Determining Densities and Energy Levels of Dopants and Traps by Means of Hall-Effect Measurement

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(Received March 23, 1999; accepted for publication June 25, 1999)

Using the temperature dependence $p(T)$ of the majority-carrier concentration in a semiconductor, two graphical methods for determining the densities and energy levels of dopants and traps are compared. In the analysis proposed previously, the function to be evaluated is defined by $S(T, E_{\rm ref}) \equiv p(T)\exp(E_{\rm ref}/kT)/kT$, where T is the temperature, k is the Boltzmann constant and $E_{\rm ref}$ is the parameter. However, it is sometimes difficult, particularly in high-resistivity semiconductors, to determine these values. This is why the function to be evaluated is introduced here as $H(T, E_{\rm ref}) \equiv p(T)^2\exp(E_{\rm ref}/kT)/(kT)^{2.5}$, and it is determined that $H(T, E_{\rm ref})$ is superior to $S(T, E_{\rm ref})$.

KEYWORDS: dopant, trap, determination of densities and energy levels, high-resistivity semiconductor, Hall-effect measurement, temperature dependence of majority-carrier concentration, improved analysis, graphical approach

The temperature dependence $p(T)$ of the majority-carrier concentration in a semiconductor reflects important information on dopants and traps. Although there are some methods for determining the densities and energy levels of dopants, it is difficult to evaluate them accurately.[1–4] Therefore, a new method was proposed and tested experimentally.[1–3, 5–7]

In this method,[1–3, 5–7] the function to be evaluated is defined by

$$S(T, E_{\rm ref}) \equiv \frac{p(T)}{kT}\exp\left(\frac{E_{\rm ref}}{kT}\right), \tag{1}$$

where T is the temperature, k is the Boltzmann constant and $E_{\rm ref}$ is the parameter. $S(T, E_{\rm ref})$ has its peak at a temperature corresponding to each dopant level or each trap level. Thus, using each peak temperature and peak value, the corresponding energy level and density can be accurately determined.

This analysis was applied to p-type silicon (p-Si) irradiated with a low fluence ($< 1\times10^{14}$ cm^{-2}) of 10-MeV protons,[2] and nitrogen (N)-doped 4H-SiC,[3] and then reliable values were obtained in these low-resistivity semiconductors. On the other hand, the method could not be applied to p-Si irradiated with a high fluence ($\geq 1\times10^{14}$ cm^{-2}) of 10-MeV protons, because the resistivity of p-Si increased following high-fluence irradiation. This is why an improved function $H(T, E_{\rm ref})$ is proposed instead of $S(T, E_{\rm ref})$.

Let us consider one type of acceptor (acceptor density $N_{\rm A}$ and acceptor level $E_{\rm A}$) and one type of donor (donor density $N_{\rm D}$) in a p-type semiconductor. From the charge neutrality condition, $p(T)$ is obtained as

$$p(T) = N_{\rm A} f_{\rm A}(E_{\rm A}) - N_{\rm D}, \tag{2}$$

where $f_{\rm A}(E)$ is the Fermi-Dirac distribution function for acceptors. Thus, $S(T, E_{\rm ref})$ is expressed as

$$S(T, E_{\rm ref}) = \frac{N_{\rm A}}{kT}\exp\left[-\frac{(E_{\rm A}-E_{\rm V})-E_{\rm ref}}{kT}\right]F_{\rm A}(T) - \frac{N_{\rm D}}{kT}\exp\left(\frac{E_{\rm ref}}{kT}\right) \tag{3}$$

with

$$F_{\rm A}(T) = \frac{\exp\left(\dfrac{E_{\rm F}-E_{\rm V}}{kT}\right)}{g_{\rm A}+\exp\left(\dfrac{E_{\rm F}-E_{\rm A}}{kT}\right)}, \tag{4}$$

where $E_{\rm F}$ is the Fermi level, $E_{\rm V}$ is the energy level at the top of the valence band and $g_{\rm A}$ is the degeneracy factor of acceptors.

The function

$$\frac{N_{\rm A}}{kT}\exp\left[-\frac{(E_{\rm A}-E_{\rm V})-E_{\rm ref}}{kT}\right], \tag{5}$$

in eq. (3), has a peak value of $N_{\rm A}\exp(-1)/kT_{\rm peak}$ at $T_{\rm peak} = [(E_{\rm A}-E_{\rm V})-E_{\rm ref}]/k$. Even when no peak appears in the measured temperature range with $E_{\rm ref} = 0$ eV, furthermore, the peak temperature of $S(T, E_{\rm ref})$ can be shifted to the measured temperature range by changing $E_{\rm ref}$. To determine reliable values of $N_{\rm A}$ and $E_{\rm A}$ using the peak value and peak temperature, $F_{\rm A}(T)$ should be less dependent on T.

To obtain a function less temperature dependent than $F_{\rm A}(T)$, the function to be evaluated is defined by

$$H(T, E_{\rm ref}) \equiv \frac{p(T)^2}{(kT)^{2.5}}\exp\left(\frac{E_{\rm ref}}{kT}\right). \tag{6}$$

Substituting the $p(T)$ in eq. (2) for one of the $p(T)$ in eq. (6) and substituting

$$p(T) = N_{\rm V}(T)\exp\left(-\frac{E_{\rm F}-E_{\rm V}}{kT}\right), \tag{7}$$

for the other $p(T)$ in eq. (6), $H(T, E_{\rm ref})$ gives

$$H(T, E_{\rm ref}) = \frac{N_{\rm A}}{kT}\exp\left[-\frac{(E_{\rm A}-E_{\rm V})-E_{\rm ref}}{kT}\right]I_{\rm A}(T) - \frac{N_{\rm D}}{kT}\cdot N_{\rm V0}\exp\left[\frac{E_{\rm ref}-(E_{\rm F}-E_{\rm V})}{kT}\right], \tag{8}$$

where

$$I_{\rm A}(T) = \frac{N_{\rm V0}}{g_{\rm A}+\exp\left(\dfrac{E_{\rm F}-E_{\rm A}}{kT}\right)}, \tag{9}$$

and $N_{\rm V}(T)$ is the effective density of states in the valence band, which is given by[8]

$$N_{\rm V}(T) = N_{\rm V0}k^{1.5}T^{1.5}, \tag{10}$$

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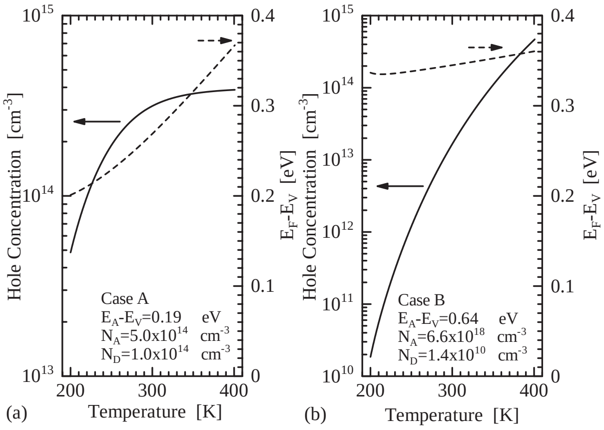

Fig. 1. $p(T)$ and $E_F - E_V$ simulated for (a) Case A and (b) Case B.

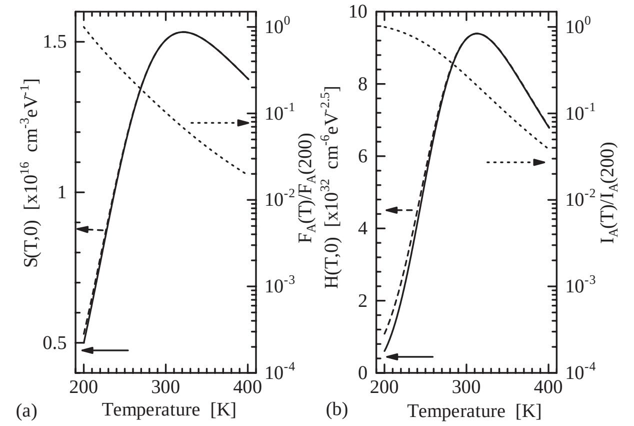

Fig. 2. $S(T, 0)$, $F_A(T)/F_A(200)$, $H(T, 0)$ and $I_A(T)/I_A(200)$ for Case A.

and N_{V0} is a constant. Since the numerator of $I_A(T)$ is constant, $I_A(T)$ is considered to be less temperature dependent than $F_A(T)$.

To compare $I_A(T)$ with $F_A(T)$ from the viewpoint of temperature dependence, two types of high-resistivity p-Si are considered: (1) Case A with $E_A - E_V = 0.19\,\text{eV}$, $N_A = 5.0 \times 10^{14}\,\text{cm}^{-3}$ and $N_D = 1.0 \times 10^{14}\,\text{cm}^{-3}$, and (2) Case B with $E_A - E_V = 0.64\,\text{eV}$, $N_A = 6.6 \times 10^{18}\,\text{cm}^{-3}$ and $N_D = 1.4 \times 10^{10}\,\text{cm}^{-3}$. Figure 1 shows $p(T)$ and $E_F - E_V$ simulated using eqs. (2) and (7) for both cases.

Figure 2 shows $S(T, 0)$, $F_A(T)/F_A(200)$, $H(T, 0)$ and $I_A(T)/I_A(200)$ for Case A. The solid line of $S(T, 0)$ or $H(T, 0)$ is calculated from eq. (1) or eq. (6). On the assumption that $N_D = 0\,\text{cm}^{-3}$, N_A and E_A are determined using the peak temperature and the value in the solid line. Using the obtained values ($E_A - E_V = 0.19\,\text{eV}$ and $N_A = 4.0 \times 10^{14}\,\text{cm}^{-3}$), the broken line of $S(T, 0)$ or $H(T, 0)$ is simulated using eq. (3) or eq. (8), and the dotted line of $F_A(T)/F_A(200)$ or $I_A(T)/I_A(200)$ is simulated using eq. (4) or eq. (9).

The figure indicates that $I_A(T)$ is less temperature dependent than $F_A(T)$. As is clear from eq. (3) or eq. (8), N_D is determined from the difference between the solid and broken lines in $S(T, E_{\text{ref}})$ or $H(T, E_{\text{ref}})$. Since this difference in $H(T, E_{\text{ref}})$ is larger than that in $S(T, E_{\text{ref}})$, N_D as well as N_A and E_A can be determined more reliably in $H(T, E_{\text{ref}})$ than in $S(T, E_{\text{ref}})$.

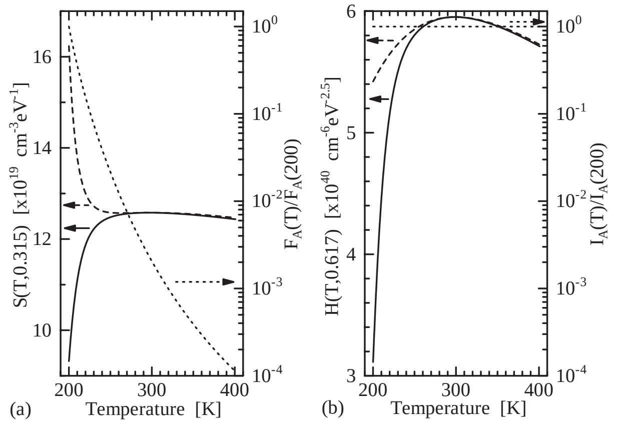

Fig. 3. $S(T, 0.315)$, $F_A(T)/F_A(200)$, $H(T, 0.617)$ and $I_A(T)/I_A(200)$ for Case B.

In Fig. 3, $S(T, 0.315)$, $F_A(T)/F_A(200)$, $H(T, 0.617)$ and $I_A(T)/I_A(200)$ for Case B are shown. The solid line of $S(T, 0.315)$ or $H(T, 0.617)$ is calculated from eq. (1) or eq. (6). On the assumption that $N_D = 0\,\text{cm}^{-3}$, N_A and E_A are determined using the peak temperature and the value in the solid line of $H(T, 0.617)$. Using the obtained values ($E_A - E_V = 0.64\,\text{eV}$ and $N_A = 6.7 \times 10^{18}\,\text{cm}^{-3}$), the broken line of $S(T, 0.315)$ or $H(T, 0.617)$ is simulated from eq. (3) or eq. (8), and the dotted line of $F_A(T)/F_A(200)$ or $I_A(T)/I_A(200)$ is simulated using eq. (4) or eq. (9).

As is clear from the figure, $I_A(T)$ is independent of T, while $F_A(T)$ varies with T. The broken line of $S(T, 0.315)$ does not exhibit any peaks because $F_A(T)$ decreases with T too rapidly. As a result, E_A, N_A and N_D cannot be determined. On the other hand, since the broken line of $H(T, E_{\text{ref}})$ has a peak, E_A, N_A and N_D can be determined accurately.

In summary, compared with $S(T, E_{\text{ref}})$ proposed previously, $H(T, E_{\text{ref}})$ is more suitable for determining the densities and energy levels using $p(T)$, since $I_A(T)$ included in $H(T, E_{\text{ref}})$ is much less temperature dependent than $F_A(T)$ included in $S(T, E_{\text{ref}})$.

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